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VOTING FOR PUBLIC ALTERNATIVES:

SOME NOTES ON MAJORITY RULE*

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Voting for Public Alternatives: Some Notes on Majority Rule

Majority voting is an institution common to almost all democratic societies, but, as a method of social decision-making, it is vulnerable to attack on several fronts. In this paper I examine some of the well-known shortcomings of majority voting: intransitivity, indecisiveness, and susceptibility to strategic manipulation. Most important, I consider the assertion that majority rule is Pareto inefficient and show that this inefficiency is related to the shape of individuals' utility functions. I also present a result which enables one to predict the direction in which the inefficiency lies. Finally, I show that rank-order voting is likely to improve efficiency.

One weakness of majority voting, first enunciated by Condorcet, is its failure, in general, to generate a transitive social ordering of alternatives. Given three alternatives, a, b, and c, a majority of the electorate may prefer a to b and b to c and yet c to a.¹ This flaw is far from unique to decision-making by majority rule. As Arrow [1951] has demonstrated, the problem plagues any social choice procedure that satisfies several reasonable conditions.

More seriously, majority voting may be non-decisive; a vote may fail to produce a majority winner at all. Non-decisiveness has two possible senses. One is simply that, with more than two candidates on the ballot, none may capture over fifty percent of the vote. Thus, there was concern in the 1968 U.S. Presidential election that George Wallace's presence would prevent either of the real contenders, Richard Nixon and Hubert Humphrey, from obtaining a majority.

A second interpretation is that no candidate may emerge victorious even in pair-wise competition against the other alternatives. A pair-wise majority winner was presumably not absent in the 1968 election. That is, most likely Nixon would have won a majority in head-to-head competition against either Humphrey or Wallace, even had he not obtained a majority in the three-way race. In practice, the contingency of no majority winner (in either sense) is often handled by declaring the plurality winner the victor or by holding a run-off election between the two most popular candidates of the initial election. Both these procedures, however, have drawbacks discussed below.

The standard response to the contention that majority rule leads to intransitivity and non-decisiveness is to note that in many important applications, individuals' preferences are likely to be single-peaked. As Black [1948], anticipated by Bowen [1942], shows, the existence of a natural ordering of alternatives under which single-peakedness obtains guarantees both the transitivity of social preferences and the existence of a pair-wise majority winner. I should point out, however, that single peakedness does not ensure a majority winner in the first sense in elections with three or more candidates. For example, suppose that candidates are naturally ordered according to their liberalism or conservatism. Consider an election with three candidates: a right-winger (R), a moderate (M), and a left-winger (L). The following preferences are single-peaked: (R, M, L), (M, R, L), and (L, M, R), where alternatives are listed in order of decreasing preference. Suppose that the preferences among the electorate divide, with (R, M, L) accounting for forty-eight percent, (M, R, L) for three percent, and (L, M, R) for forty-nine percent. Notice, that, in pair-wise competition, M would

win, with R finishing second. In the three-way race, however, not only is there no majority winner if individuals vote for their favorite candidate, but the order of finish is exactly the opposite of the "true" (i.e., pair-wise majority) social ordering.

The example demonstrates, therefore, that plurality rule - used in congressional elections in many states - can lead to highly perverse outcomes even when preferences are single-peaked. Indeed, Weber [1978] has noted that in the three-way 1970 Senate race in New York, the victor, James Buckley, almost surely would have finished last in pair-wise comparisons. A multi-candidate election followed by a two candidate runoff - used, for example, to select the president of France - also fails, in general, to elect the pair-wise majority winner; indeed, it fails in the above example. One expects this method, however, to lead to less perverse outcomes than does plurality rule. In the New York election, for instance, Buckley would probably have lost a runoff. The method is, nonetheless, not infallible.

The failure of two common voting procedures to elect majority winners, even when preferences are single-peaked, prompts the question of whether some other method might succeed. One natural procedure is to hold a series of pair-wise elections, with the majority winner of any given election paired against some yet untried alternative in the next round. That alternative which remains undefeated until the end is elected. It is easy to see that if individuals always vote sincerely - if in each election they always vote for the alternative higher in their preference ordering - then the pair-wise majority winner will emerge the victor.

Unfortunately, there may be strong incentives for insincere voting. Consider the above three candidate example again. Suppose that in round one, alternative M is paired against L, with the winner facing R in round 2. With sincere voting, M would defeat L and R, in turn. In the first round, however, an individual with preferences (R, M, L) might foresee that L will inevitably lose. With this understanding, he has the incentive to vote for L rather than M, in the hope that L will oppose R in round 2. If all forty-eight percent of the electorate preferring R vote accordingly (and if the rest of the population votes sincerely) R will be the ultimate winner. Sequential voting, therefore, may not work.

There is, however, a simple election procedure, which selects majority winner and is immune to incentives for misrepresentation, provided that preferences are single peaked and that the "natural" ordering of alternatives is known. In the scheme, individuals vote for one candidate, and the median candidate among the votes cast is declared the winner.² Thus, in our political example, with 48 votes for R, 3 votes for M and 49 votes for L, M would be the median candidate on the conservative-liberal scale and would, therefore be elected. Notice that no individual acting alone or in collusion with others has any inducement to vote for any candidate other than his favorite. If an individual's favorite lies, say, to the left of the (sincere) median, he can alter the outcome of the election through misrepresentation only by voting for a candidate lying to the right. Since such behavior can move the outcome only further to the right, it is, therefore, clearly self-defeating. Individuals should, therefore, vote sincerely. Since this argument holds equally well for coalitions, I conclude that this method of selecting majority winners³ is coalitionally strategy-proof in the sense of

Gibbard [1973] and Satterthwaite [1975].

There is, however, an important qualification to the assertion of majority rule's strategy-proofness. The above argument implicitly ruled out sidepayments (bribes) among individuals. If sidepayments were possible, strategy-proofness would break down because then individuals could be compensated for voting for non-favorite candidates.

The issue of sidepayments is closely related to the question of majority rule's efficiency. That majority rule is Pareto inefficient has been, at least among economists, its severest criticism. Of course, in one sense, a majority winner is, almost by definition, Pareto optimal; no other alternative can be unanimously preferred. If, however, one also allows for transfers of some private good or money among voters, efficiency vanishes. With transfers, one must redefine a Pareto optimum as a public alternative which is not Pareto-dominated by the combination of another alternative and a system of transfers. It should be clear that one cannot expect majority rule to be efficient, in general, under this new definition. The problem is that majority rule, which is defined purely in terms of ordinal comparisons cannot capture strengths of preferences for public alternatives as measured by equivalent quantities of private goods.

Of course, when economists complain of an inefficient majority outcome, they ordinarily do not seriously propose that the sidepayments actually be made.⁴ Concern about efficiency, as Weber [1978] has suggested, derives from arguments similar to the justification by Harsanyi [1955] of utilitarianism. If an individual knows that he is to participate in a stream of future elections but does not know what the issues will be, then, as an expected utility maximizer, he is best off ex ante under a voting scheme which yields Pareto optimal outcomes.

Although majority rule is, in general, inefficient, Bowen (1942] and Bergstrom [1978] have demonstrated that if preferences for public alternatives are, in some sense, symmetrically distributed across society, then majority winners are Pareto optimal after all. I wish to show that the same conclusion can be drawn by substituting, for symmetry, an assumption about the shape of individual's preferences. Suppose that public alternatives can be associated with numbers in the unit interval $[0, 1]$. Suppose, furthermore, that an individual's utility (say, in dollars) declines linearly (and with the same slope in both directions) as the public alternative moves away from his favorite alternative. That is, his dollar utility function is of the form

$$(1) \quad u(x) = c - b|x - a|,$$

where x is the public alternative, a is the individual's favorite alternative, b is a positive constant, and c is a constant. An individual can thus be summarized by the parameters a , b , and c . Let $F(a, b, c)$ be the joint distribution of characteristics across society. I assert the following:

Proposition 1:⁵ If individuals' utilities are of the form (1) and if the distribution of a is independent of that of b , then the majority winner is Pareto optimal.

Proof: A Pareto optimum is an alternative, \bar{x} , such that

$$\int (c - b|\bar{x} - a|) dF(a, b, c) = \max_x \int (c - b|x - a|) dF(a, b, c).$$

Since the parameter c is clearly irrelevant for efficiency, we can suppress it. We have therefore,

$$-\int b|\bar{x}-a|dF(a, b) = \max_x (-\int b|x-a|dF(a, B)) = \min_x \int b|x-a|dF(a, b).$$

From the independence of a and b , $\min_x \int b|x-a|dF(a, b) = \bar{b} \min_x \int |x-a|df(a)$,

where \bar{b} is the mean of the marginal distribution of b , and f is the marginal cumulative distribution function for a . But, it is well known that

$$\min_x \int |x-a|df(a) = \int |a^*-a|df(a), \text{ where } a^* \text{ is the median of the distribution } f.$$

Since the median of f is the majority winner, we conclude that majority rule is Pareto efficient.

Q.E.D.

Proposition 1 is of interest less for its own sake than for the light it sheds on the direction of bias away from efficiency that one can expect with majority rule. Proposition 1 depends on the magnitude of an individual's marginal utility remaining a constant, b , whether x is close to or far from his favorite alternative a . It is probably far more plausible to suppose, however, that the magnitude of marginal utility is greater the further x is from a . Indeed, if utility is differentiable, then, in the neighborhood of a , marginal utility is nearly zero.

Specifically, I shall assume that preferences are distributed so that, for two populations of equal size, the sum of the magnitudes of the marginal utilities is greater for that population the mean of whose favorite alternatives is farthest from the chosen public alternative. I then can state the following simple result.

Proposition 2: If preferences satisfy the property of the preceding paragraph, and if utility functions are concave, then if the median of the distribution of a 's exceeds the mean, the majority winning public alternative exceeds the Pareto optimum. Similarly, if the median of the

a's is less than the mean, the majority winner will be less than the Pareto optimum.

Proof: Suppose that the median of the a's, a^* , is greater than the mean. (The argument is symmetric for a^* less than the mean.) Suppose the public alternative is slightly diminished from the level a^* . By definition of a^* , the populations of individuals whose a's lie above and below a^* are of equal size. By hypothesis, the mean of the a's less than a^* is farther from a^* than the mean of the a's greater than a^* . Thus, also by hypothesis, the small decrease in public alternative must raise the sum of utilities. Since utility functions are concave, we can conclude that the Pareto optimum is less than a^* .

Q.E.D.

Proposition 2 suggests that if the distribution of a's is skewed to the right, majority rule will yield outcomes which are too large relative to efficiency, while a skew to the left leads to outcomes that are too small. In other words, majority decisions tend to be too extreme. As I mentioned above, majority rule's failure to attain efficiency is due to its inability to take preference intensities into account. One may ask, therefore, if other methods of decision-making perform better.

One familiar alternative to majority rule is the method of rank-order voting, sometimes called the Borda count. Under this scheme, if there are m candidates, a voter assigns to these candidates the weights $1, \dots, m$ in order of increasing preference, and the candidate receiving the highest sum of weights wins. It is obvious that if $m=2$, the Borda count and majority rule are identical, but for $m \geq 3$, the former seems to

capture some degree of preference strength which the latter cannot.

I should like to argue now that rank-order voting tends to produce more "moderate" outcomes.

Suppose that the candidates are x , y , and z with y the Pareto optimal alternative and $x < y < z$. From proposition 2, we expect that if the distribution of a 's is skewed enough, an alternative other than y will emerge as the majority winner. Suppose, in fact, that z is the majority winner; i.e., the median of the a 's lies to the right of y . Under rank-order voting, z receives 3 votes from each individual whose a is greater than the median, while y receives two votes. That is, z picks up one net vote over y for each a to the right of the median. On the other hand, y picks up at least one vote for each a to the left of the median, and, for those individuals whose preferences are (y, x, z) , two votes. Thus, under rank-order voting, there is a natural bias in favor of alternative y , the Pareto optimum. The Borda count is, in this sense, more likely than majority rule to result in Pareto efficiency.

FOOTNOTES

¹Consider three individuals, 1, 2, and 3. If 1 prefers a to b to c, 2, b to c to a, and 3, c to a to b, a cyclic majority results. This is the famous "paradox of voting".

²T. Bergstrom has informed me that this method of voting is actually used by some communities to decide referenda.

³This is by no means the only strategy-proof procedure for selecting majority winners, although I cannot think of a simpler one. An alternative, but more complicated, scheme has people vote by announcing preference orderings (under the constraint that the orderings must be single-peaked). The majority winner can then be computed directly from the announcement.

⁴There are some exceptions. Log-rolling in legislatures is an implicit form of sidepayment.

⁵I am grateful to Martin Weitzman, who inspired this result by reminding me of similar proposition.

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